647510 **BSc6MatC602x** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
March/April-2023 (NEW COURSE)
Mathematical Analysis - 2 and Abstract Algebra-2 (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following question. (Any Two) **(10)**

1. Show that continuous image of compact set is compact.
2. State and prove theorem of nested interval.
3. Prove that every compact set of a metric space is closed and bounded.

Q.1(B) Answer the following questions. (Any One) **(04)**

1. Show that set of all real number is uncountable.
2. Check whether the following subsets of R are Compact and connected. Give reason for your answer. **(i)** R **(ii)** $(0,2] \cap [2,5)$

Q.2(A) Answer the following question. (Any Two) **(10)**

1. if f(t) is continuous for all $t \geq 0$ and f'(t) is piecewise Continuous then prove that L{f'(t)} Exists, provided $\lim_{n\to\infty} e^{-st} f(t) = 0$ and L{f'(t)}=sL{f(t)}-f(0)=s $\bar{f}$ (s)-f(0). Also find L[t^n] where n is non negative integer.
2. State and prove change the scale property. Using this property find L[sinh(at) sin(at)].
3. Find : L[t cosh(at)] using derivative of Laplace Transform.

Q.2(B) Answer the following questions. (Any One) **(04)**

1. Find : $L^{-1}[\frac{4s+5}{(s-1)^2(s-2)}]$
2. Find : $L^{-1}[\frac{1}{s^4-2s^3}]$

Q.3(A) Answer the following question. (Any Two) **(10)**

1. State convolution theorem and Using this theorem Find
$$L^{-1}[\frac{1}{s^2(s-1)}].$$
2. Prove that if L{f(t)}= $\bar{f}(s)$ then $L[\int_0^t f(u)du] = \frac{1}{s}\bar{f}(s)$.

   Using this result Find $L^{-1}[\frac{1}{s^3(s^2+a^2)}]$.
3. If L{f(t)}= $\bar{f}(s)$ and $\frac{f(t)}{t}$ has laplace transform then prove

   That $L\{\frac{f(t)}{t}\} = \int_s^\infty \bar{f}(s)ds$. Using this result Find $L[\frac{\sin(t)}{t}]$.

Q.3(B) Answer the following questions. (Any One) **(04)**

1. By using integration of laplace transform find $L^{-1}[\frac{1}{s(s+a)^3}]$.
2. Find : $L[e^{-4t}\int_0^t t\sin(3t)dt]$.

Q.4(A) Answer the following question. (Any Two) **(10)**

1. Define kernel of homomorphism. If $\phi: G \rightarrow G'$ is homomorphism Then prove that $K(\phi)$ is a normal subgroup of G.
2. Prove that characteristic of integral domain is either zero or prime number.
3. State and prove fundamental theorem of Homomorphism.

Q.4(B) Answer the following questions. (Any One) **(04)**

1. For non zero polynomials f(x),g(x) of integral domain Show that [f(x)g(x)]=[f(x)]+[g(x)].
2. If P is prime number then show that Z_p is field.

Q.5(A) Answer the following question. (Any Two) **(10)**

1. State and prove division algorithm theorem of polynomial.
2. Show that factors of $x^4+3x^3+2x+4=0$ in $Z_5[X]$ are $(x-1)^3(x+1)$.
3. Define irreducible polynomial and show that $f(x)= x^4-2x^2+8x+1$ is irreducible over Q.

Q.5(B) Answer the following questions. (Any One) **(04)**

1. Define monic polynomial. Find G.C.D. d(x) of f(x),g(x) $\in$ $Z_5[X]$. Where $f(x)=x^3+3x^2+3x+3$ and $g(x)=4x^3+2x^2+2x+2$. also express d(x) as a(x)f(x)+b(x)g(x).
2. If $a=a_1+a_2i+a_3j+a_4k$ is any non zero quaternion number then Find a^{-1}.
   Check whether quaternion set Q is Field.

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